

7-6 Rational Zero Theorem

Solve, given that -4 is a zero.

$$x^3 - 11x + 20 = 0$$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -11 & 20 \\ & & -4 & 16 & -20 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$$x^2 - 4x + 5$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2}$$

$$\frac{4 \pm 2i}{2}$$

$$\{-4, 2 \pm i\}$$

Rational Zero Theorem-- $\frac{p}{q}$ is a possible zero, where:

- $p \in$ set of integral factors of the constant
- $q \in$ set of the integral factors of the leading coefficient

Solve

$$0 = 6x^3 + 7x^2 - 9x + 2$$

$$p \in \{\pm 1, \pm 2\}$$

$$q \in \{\pm 1, \pm 2, \pm 3, \pm 6\}$$

$$\frac{p}{q} \in \left\{ \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3} \right\}$$

$$\begin{array}{r|rrrr} -2 & 6 & 7 & -9 & 2 \\ & & -12 & 10 & -2 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

$$6x^2 - 5x + 1$$

$$6x^2 - 2x - 3x + 1$$

$$2x(3x-1) - 1(3x-1)$$

$$(2x-1)(3x-1) = 0$$

$$\left\{ -2, \frac{1}{3}, \frac{1}{2} \right\}$$

Solve

$$x^4 - x^3 + 7x^2 - 9x - 18 = 0$$

$$P \in \{\pm 1 \pm 2 \pm 3 \pm 6 \pm 9 \pm 18\}$$

$$q \in \{\pm 1\}$$

$$\frac{P}{q} \in \{$$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 7 & -9 & -18 \\ & & -1 & 2 & -9 & 18 \\ \hline 2 & 1 & -2 & 9 & -18 & 0 \\ & & 2 & 0 & 18 & \\ \hline & 1 & 0 & 9 & 18 & \end{array}$$

$$\{-1, 2, \pm 3i\} \quad \begin{aligned} x^2 + 9 &= 0 \\ x^2 &= -9 \\ x &= \pm 3i \end{aligned}$$

Do:

$$0 = 2x^4 + 3x^3 + 6x^2 + 12x - 8$$

Solve.

$$0 = x^4 - 4x^3 + 6x^2 - 8x + 8$$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 6 & -8 & 8 \\ & & 2 & -4 & 4 & -8 \\ \hline 2 & 1 & -2 & 2 & -4 & 0 \\ & & 2 & 0 & 4 & \\ \hline & 1 & 0 & 2 & 4 & \end{array}$$

$$x^2 = -2$$

$$x = \pm i\sqrt{2}$$

$$\{2, 2, \pm i\sqrt{2}\}$$

Solve.

$$0 = x^4 - 6x^3 + 8x^2 - 48x$$

Factor out GCF

$$0 = x(x^3 - 6x^2 + 8x - 48)$$

$$x = 0$$

$$p \in \{\pm 1 \pm 2 \pm 3 \pm 4 \pm 6 \pm 8 \pm 12\}$$

$$q \in \{\pm 1\} \quad \pm 16 \pm 24 \pm 48\}$$

$$\begin{array}{r|rrrr} 6 & 1 & -6 & 8 & -48 \\ & & 6 & 0 & 48 \\ \hline & 1 & 0 & 8 & 10 \end{array}$$

$$x^2 + 8 = 0$$

$$\{0, 6, \pm 2i\sqrt{2}\}$$

HW

p381

13, 15, 19, 25, 28, 30